EXAMPLE – 2

1) i) Transform the equation $x^2+y^2-4x+8y-17=0$ to parallel axes through the point $(2,-4)$

Solution:

given equation is
$x^2+y^2-4x+8y-17=0$ —— (1)

Let us take O as origin and OX, OY as co-ordinate axes.
Suppose the origin be shifted to the point $O(+2,-4)$ the new axes becomes $O_1X_1$, $O_1Y_1$. We have

$x = x_1+h$ & $y = y_1+k$
$= x_1+2$ $= y_1-4$.

Putting the value of x and y in eqⁿ (1) we get.

$(x_1+2)^2+(y_1-4)^2-4(x_1+2)+8(y_1-4)-17=0$

or, $x_1^2+\cancel{4x_1}+4+y_1^2+16-\cancel{8y_1}-\cancel{4x_1}-8+\cancel{8y_1}$
$-32-17=0$.

or, $x_1^2+y_1^2-37=0$

∴ Thus, the new transformed eqⁿ is
$\boxed{x_1^2+y_1^2-37=0}$.

ii) Transform the equation $x^2-3y^2+4x+6y=0$ to parallel axes through the point $(-2,1)$.

Solution:

given equation is
$x^2-3y^2+4x+6y=0$ ——— (1)

let us take $O$ as origin and $OX$ and $OY$ as co-ordinate axes.
Suppose the origin be shifted to the point $(-2,1)$ the new axes becomes $O_1X_1$, $O_1Y_1$
we have
$x = x_1+h$
$= x_1-2$

& $y = y_1+k$
$= y_1+1$

putting the value of $x$ and $y$ in eq$^n$ (1)
we get
$(x_1-2)^2-3(y_1+1)^2+4(x_1-2)+6(y_1+1)=0$
or, $x_1^2+4-4x_1-3(y_1^2+1+2y_1)+4x_1-8+6y_1+6=0$
or, $x_1^2+4-3y_1^2-3-\cancel{6y_1}-8+\cancel{6y_1}+6=0$

or, $x_1^2-3y_1^2-1=0$

∴ Thus, the new transformed eq$^n$ is
$\boxed{x_1^2-3y_1^2=1}$

iii) what does the equation $x^2-y^2+2x+4y=0$ become when the origin is transformed to the point $(-1,2)$ the directions of axes remaining unchanged?

Solution:

given equation is

$x^2-y^2+2x+4y=0$ —— (1)

Let us take O as origin and OX and OY as coordinate axes.

Suppose the origin be shifted to the point $(-1,2)$ the new axes becomes $O_1X_1$ & $O_1Y_1$

we have

$x = x_1+h$

$= x_1-1$

&

$y = y_1+k$

$= y_1+2$

putting the value of x and y in eq$^n$ (1) we get

$(x_1-1)^2-(y_1+2)^2+2(x_1-1)+4(y_1+2)=0$

or, $x_1^2-\cancel{2x_1}+1-y_1^2-4-\cancel{4y_1}+\cancel{2x_1}-2+\cancel{4y_1}+8=0$

or, $x_1^2-y_1^2+3=0$

Thus, the new transformed eq$^n$ is

$\boxed{x_1^2-y_1^2+3=0}$

iv) What does the equation $(a-b)(x^2+y^2)-2abx=0$ become if the origin be moved to the point $\left(\frac{ab}{a-b}, 0\right)$?

Solution:

given equation is

$(a-b)(x^2+y^2) - 2abx = 0$

Let us take O as origin and OX and OY as coordinate axes.

Suppose the origin be shifted to the point $\left(\frac{ab}{a-b}, 0\right)$ the new axes becomes $O_1X_1$

& $O_1Y_1$ we have

$x = x_1 + h$

$= x_1 + \frac{ab}{a-b}$

&

$y = y_1 + K$

$= y_1 + 0$

putting the value of x and y in eqn ① we get

$(a-b)\left\{\left(x_1 + \frac{ab}{a-b}\right)^2 + y_1^2\right\} - 2ab\left(x_1 + \frac{ab}{a-b}\right) = 0$

or, $(a-b)\left[\left\{\frac{(a-b)x_1 + ab}{a-b}\right\}^2 + y_1^2\right] - 2abx_1 - \frac{2a^2b^2}{a-b} = 0$

or, $(a-b)\left[\frac{(a-b)^2x_1^2 + a^2b^2 + 2abx_1(a-b)}{(a-b)^2} + y_1^2\right] - 2abx_1$

$- \frac{2a^2b^2}{a-b} = 0$

or, $(a-b)\left[\frac{(a-b)^2x_1^2 + a^2b^2 + 2abx_1(a-b) + (a-b)^2y_1^2}{(a-b)^2}\right] - 2abx_1 - \frac{2a^2b^2}{a-b} = 0$

or, $\frac{(a-b)^2x_1^2 + a^2b^2 + 2abx_1(a-b) + (a-b)^2y_1^2}{(a-b)} - 2abx_1 - \frac{2a^2b^2}{a-b} = 0$

or, $(a-b)x_1^2 + \frac{a^2b^2}{a-b} + \cancel{2abx_1} + (a-b)y_1^2 - \cancel{2abx_1} - \frac{2a^2b^2}{a-b} = 0$

or, $(a-b)x_1^2 + (a-b)y_1^2 - \frac{a^2b^2}{a-b} = 0$

or, $(a-b)^2x_1^2 + (a-b)^2y_1^2 - a^2b^2 = 0$

or, $(a-b)^2(x_1^2 + y_1^2) - a^2b^2 = 0$

Thus, the new transformed equation is

$$\boxed{(a-b)^2(x_1^2 + y_1^2) = a^2b^2}$$

v) Transform to parallel axes through the point $(-2,3)$ the equation
$2x^2 + 4xy + 5y^2 - 4x - 22y + 7 = 0$

Solution:

given equation is
$2x^2 + 4xy + 5y^2 - 4x - 22y + 7 = 0$ —— (1)

Let us take O as origin and OX and OY as coordinate axes.
Suppose the origin be shifted to the point (-2,3) the new axes becomes $O_1X_1$ & $O_1Y_1$
we have

$x = x_1 + h$
$= x_1 - 2$

&

$y = y_1 + k$
$= y_1 + 3$

putting the value of x and y in eqⁿ ① we get

$2(x_1-2)^2 + 4(x_1-2)(y_1+3) + 5(y_1+3)^2 - 4(x_1-2) - 22(y_1+3) + 7 = 0$

or, $2(x_1^2 - 4x_1 + 4) + 4(x_1y_1 + 3x_1 - 2y_1 - 6) + 5(y_1^2 + 6y_1 + 9) - 4x_1 + 8 - 22y_1 - 66 + 7 = 0$

or, $2x_1^2 - 8x_1 + 8 + 4x_1y_1 + 12x_1 - 8y_1 - 24 + 5y_1^2 + 30y_1 + 45 - 4x_1 + 8 - 22y_1 - 66 + 7 = 0$

or, $2x_1^2 + 5y_1^2 - 12x_1 + 12x_1 + 30y_1 - 30y_1 + 4x_1y_1 - 22 = 0$

or, $2x_1^2 + 5y_1^2 + 4x_1y_1 - 22 = 0$

Thus, the new transformed equation is

$\boxed{2x_1^2 + 5y_1^2 + 4x_1y_1 - 22 = 0}$